The Effect of Turbulence on Hot-Film Anemometer Response in Viscoelastic Fluids

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From studies of turbulence in round jets of Newtonian and non-Newtonian fluids, it has been found that the response of wedge shaped, hot-film probes can be influenced by the structure of the flow. In water and in weakly viscoelastic fluids the probe is little affected by turbulence, but for more highly viscoelastic fluids the rate of heat transfer from the hot film is markedly increased. Increases as great as 37% were found in this work; this corresponds to a 250% increase in apparent velocity. For this reason, care must be taken in interpreting measurements obtained with hot-film wedges (and probably cylinders and cones also) in flows with both laminar and turbulent regimes.

Recently, both theoretical and experimental evidence has been presented which indicates that anomalous results may be obtained when making velocity measurements in viscoelastic fluids with hot-film anemometers. Metzner and Astarita (*1*) have predicted that the heat transfer coefficient for a hot-film probe in a viscoelastic fluid will become independent of velocity at high fluid velocity. This effect, which is based on an argument that the boundary layer around a probe inserted into a viscoelastic fluid may actually begin some distance upstream of the probe (the distance increasing with velocity), would limit the hot-film anemometer to use at low velocities in such fluids. They present some experimental data for cylindrical hot film probes which is in agreement with their prediction. Fabula (*2*) has reported a similar lack of response for wedge probes in dilute Polyox solutions. Metzner and Astarita (*1*) also predict that the excessive thickness of the boundary layer on the probe would impair the response time of the probe and so would limit the use of the anemometer to the measurement of time-averaged quantities.

Smith et al. (*3*) have reported data for a cylindrical probe in dilute polymer solutions which do not show any insensitivity to velocity even at velocities up to 30 ft./sec. However, they did find that at velocities above about 20 ft./sec. (probe Reynolds number $\approx$ 250) the rate of heat transfer was unstable and would change by a factor of 3. This phenomenon disappeared as the polymer was degraded. From data taken in pipe flow experiments, these authors concluded that the response of the cylindrical probe depends not only upon the velocity and the molecular weight and concentration of the polymer in solution, but also on the ambient or free stream rate-of-strain.

In his studies of turbulence behind grids in polymer solutions, Fabula (*2*) observed a certain raggedness in his hot-film signals which he attributed to the presence of undissolved but swollen polymer particles in his solutions. This raggedness, or high wave number signal component, was observed to decrease with distance measured from the grid and was absent when the grid was removed (that is, in the laminar field).

Tennekes (*4*) and, more recently, Lumley (*5*) have considered the problem of energy and momentum transfer in the boundary layer near a wall. For the case of steady flow of dilute polymer solutions, their argument is that the dynamic relations governing the flow in this layer are altered in such a way that the viscosity sublayer is effectively thickened. The result is that the heat transfer between a heated wall and the bulk of the fluid, a process controlled by the transport, is impeded. This conclusion, of course, is in agreement with the few experimental results now available (*6*, *7*), but, as will be seen, it cannot be used to explain the measurements reported here.

For this work, wedge probes were used in two types of polymer solutions, one being viscoelastic and exhibiting drag reducing properties and the other possessing slight (if any) viscoelasticity and generally reported as ineffective in reducing drag. In neither fluid were the effects noted by Metzner and Astarita (*1*), Smith et al. (*3*), or Fabula (*2*) observed. However, in agreement with Lindgren and Chao (*6*), it was observed that the rate of heat transfer from the probe immersed in a laminar field was lower for the non-Newtonian fluids than for water when the free stream velocities were the same.

The main purpose of this paper, however, is to discuss another difficulty encountered in the use of the hot-film anemometer in turbulent flows of viscoelastic fluids, namely, the apparent dependence of its response upon the intensity of the free stream turbulence. This problem, while related to the one considered by Tennekes (*4*) and Lumley (*5*), is not quite the same since it concerns the flow near the leading edge of a wedge (the location of the heated film). Whether the same problems are encountered with cylindrical or conical probe configurations is not known; presumably they are.

EXPERIMENTAL

Experiments were carried out in submerged free jets of water, water containing various concentrations of Separan AP30 (a polyacrylamide), and water containing 0.12% of Carbopol 934 (a carboxy-vinyl polymer). The first mentioned polymer forms viscoelastic solutions, while the second exhibits practically no elastic phenomena even at the relatively high concentration employed here.

All of the Separan AP30 solutions used in the tests were prepared by diluting portions of a master batch to insure similarity of starting material. In all cases, demineralized water was used to prepare the solutions. The shear stress rate-of-strain relationships for the various solutions can be fitted by the power law relationships shown in Table I for shear rates between 1 and 800 (sec.)$^{-1}$. These data were obtained by using the Couette and cone-and-plate attachments for the Weissenberg rheogoneometer. Gupta et al (*8*) have shown that power law relationships are quite valid for these fluids over a much wider range of shear rates.

To determine the extent of the degradation of the polymer during the course of the experiments, the apparent viscosity measurements were made before and after each run. No sig-

Table 1. Rheological Data

$$\tau_{xy} = K \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y}$$

Polymer concentration, p.p.m.		K (dyne)(sec.^{n})/ sq. cm.	n
0		0.01	1.0
120	Separan AP-30	0.10	0.85
600		0.66	0.72
1,200		1.72	0.58
1,200	Carbopol 934	0.65	0.76

nificant change in the constants n and K was noted. Some degradation must have taken place, however, since a significant decrease in the stringiness (the ability of the polymer to form long stable filaments when a finger is suddenly withdrawn from the solution, presumably some measure of the elastic properties of the fluid) was noted. Interestingly, this seemed to have little affect on the measurements to be presented.

A schematic of the experimental apparatus is shown in Figure 1. The driving force for the jet was provided by a constant head tank from which the fluid flowed through a valve into a calming section provided with a series of screens and baffles. The jet was formed by forcing the fluid from the calming section through a ⅜-in. diameter square edged orifice into one end of a jet basin 6 ft. long, 2½ ft. wide, and 2½ ft. deep. At the far end of this tank the fluid flowed over a weir and thence to a pump which returned it to the head tank. The probe was mounted on a motor driven traverse assembly which allowed it to be positioned either vertically or horizontally. The entire device could be moved along the axis of the jet.

The probes employed were various size impact tubes (0.021, 0.029, 0.06, and 0.136 in. I.D.) and two glass coated hot-film wedges (sensitive element 1 mm. long by 0.2 mm. wide on an 80° wedge support).

RESULTS

Measurements in Water

The turbulent free jet has been widely studied for Newtonian fluids, and its structure is well known (9). At the origin of the jet and extending several diameters downstream is a region of potential flow in which the velocity remains essentially constant. Outside this region, in the mixing zone, turbulence is produced. This turbulence spreads both outward and inward, reaching the axis of the jet at the end of the potential core. At this point the center-line velocity begins to fall off rapidly, while the axial component of the turbulence increases greatly. The mean flow achieves its affine structure after a distance of about 10 orifice diam.

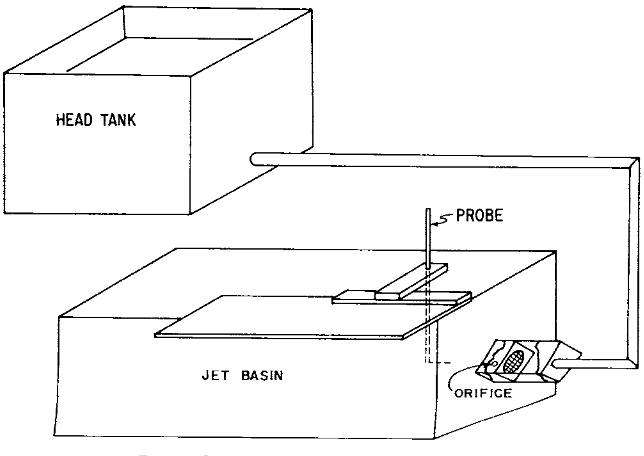

Fig. 1. Schematic of the physical system.

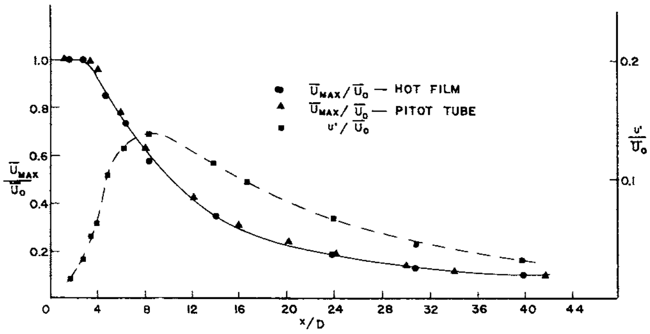

Fig. 2. Axial velocity and turbulence intensity measured along the axis of a water jet.

In this region of fully developed mean flow, the following results have been obtained from the equations of motion after the viscous terms (9) are neglected:

$$\overline{U}^{-1} \sim x + a \tag{1}$$

$$r_{1/2} \sim x + a$$

where $r_{1/2}$ is defined to be the radial distance from the jet axis at which $\overline{U}$ is equal to one-half its value on the axis. These results have been verified experimentally by many different workers (9).

The axial distribution of mean axial velocity $\overline{U}_{\max}$ (normalized with the initial velocity $\overline{U}_0$) as measured in a water jet with the Pitot tube and the hot-film anemometer is shown in Figure 2. It is seen that the length of the potential core, in this case, is between 3 and 4 diam. and that the results obtained with the two instruments are in good agreement.

The equation for the pressure P_p measured with the Pitot tube can be written as

$$P_p - p_s = \overline{\sigma_x}^* + 1/2\,\rho\overline{U^2} \tag{2}$$

For Newtonian fluids, $\overline{\sigma_x}^* = 0$ and $\overline{U^2} = \overline{U}^2 + \overline{u^2}$ (with the other components of velocity neglected). Therefore, if the total head $p_p - p_s$ is designated by $\overline{q}$, then

$$\overline{U} = \left(\frac{2\overline{q}}{\rho} - \overline{u^2} \right)^{1/2} \tag{3}$$

The velocities reported for the Pitot tube were calculated from this equation by using values of $\overline{u^2}$ measured with the hot-film anemometer. [However, Hinze (9) casts doubt on the validity of this procedure.] The values of $\sqrt{\overline{u^2}} \equiv u'$ along the jet axis are also shown in Figure 2, the values again being normalized with $\overline{U}_0$.

Pitot Tube Measurements in Viscoelastic Fluids

The interpretation of Pitot tube measurements in viscoelastic fluids is complicated by the fact that nonzero values of $\overline{\sigma_x}^*$ can occur, and these values cannot, in general, be computed (*1*, *10*, *11*). These additional normal stresses can apparently arise from two different conditions, namely, the rate of strain in the ambient or free stream and the rate of strain produced by the presence in the flow of the Pitot tube itself. On the center line of the jet, it is probable that the former effect can only be appreciable where there are large velocity gradients in the direction of the flow. The latter effect is due to the stagnation point pro-

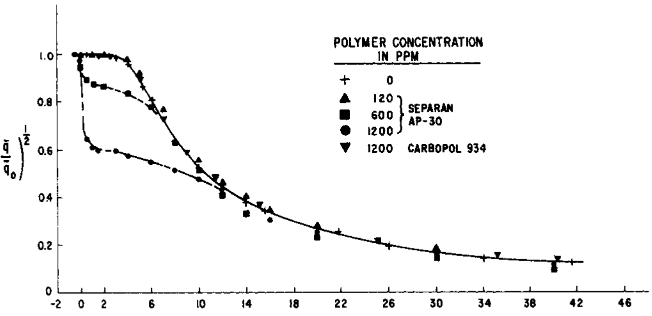

Fig. 3. Pitot tube measurements of axial velocity in jets of non-Newtonian fluids.

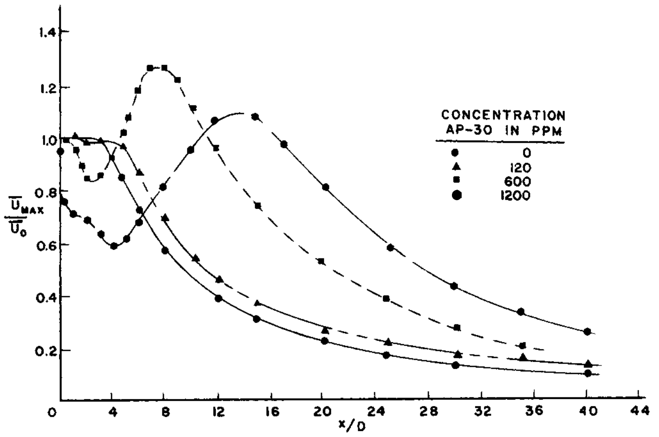

Fig. 5. Hot-film measurements of axial velocity in jets of viscoelastic fluids.

duced at the tip of the Pitot tube. Here, the axial velocity varies from zero at the center of the cross section of the tube to its free stream value at the edge of the boundary layer, that is, approximately at the wall of the tube. Thus, as pointed out by Smith et al. (3), the velocity gradient and, therefore, the magnitude of the contribution to the normal stress should be proportional to the free stream velocity and inversely proportional to the tube diameter. If this be true, then velocities calculated according to (3) from data obtained with small tubes will be considerably in error for viscoelastic fluids.

Figure 3 presents the results of Pitot tube measurements (0.136-in. tube) in three solutions of Separan AP30 having different polymer concentrations and a 1,200 p.p.m. Carbopol solution. The quantity plotted here is the square root of the total head (which is equal to $\bar{U}/\bar{U}_0$ when the effects of turbulence and normal stresses are negligible). At 120 p.p.m. Separan AP30, the data are essentially identical to the results for water. At 600 and 1,200 p.p.m., however, the curves initially decrease very rapidly, level off, and finally, further downstream, follow closely the results obtained with water. Radial profiles of the axial velocity and photographs of colored jets show that for these fluids the jet undergoes an expansion upon exiting from the orifice. Thus the initial drop in velocity for these fluids is due, at least in part, to the decrease in $\bar{U}$ resulting from this expansion. The results with Carbopol 934 are indistinguishable from those of water.

As has been pointed out, normal stress effects may also contribute to the initial drop in the axial velocity. Figure 4 shows measurements made in the 1,200 p.p.m. solution with the 0.06- and 0.136-in. tubes. Beyond about 15 diam. downstream, the two sets of measurements are in good agreement, indicating that normal stress effects, at least those which may be due to the tube size, become negligible. However, closer to the orifice, and particularly right at the origin of the flow, the differences in the two measurements are marked, indicating that normal stresses may be nonnegligible. Even larger discrepancies are obtained with still smaller tubes. In order to minimize these anomalous effects, only those measurements obtained with the largest (0.136 in.) Pitot tube have been considered in interpreting the results.

When the inverse of $\sqrt{\bar{q}}$ is plotted as a function of axial position, the linear behavior predicted by Equation (1) is obtained. The non-Newtonian fluids follow closely the line for water in the region of fully developed mean flow, although there is some indication that at 600 and 1,200 p.p.m. the curves fall off somewhat more rapidly than for water. The details of the jet measurements, including the structure of the turbulence, may be found elsewhere (12).

Hot-Film Measurements in Viscoelastic Fluids

Mean velocity measurements obtained with the hot-film anemometer in Separan AP30 solutions are shown in Figure 5. At 120 p.p.m., the mean velocity curve falls slightly below that of water at a distance of about 2 diam. downstream, rises again, and then falls off nearly parallel to, but above, the curve for water. At 600 and 1,200 p.p.m., the curves initially decrease sharply, as do the Pitot tube measurements. However, the curves then rise sharply, indicating velocities higher than the initial velocity, before finally falling off again. This latter behavior is physically unreasonable. If a momentum balance is made from velocity profiles obtained by traversing radially across the jet, the value of the integral

$$\int_0^\infty \bar{U}^2\, r dr$$

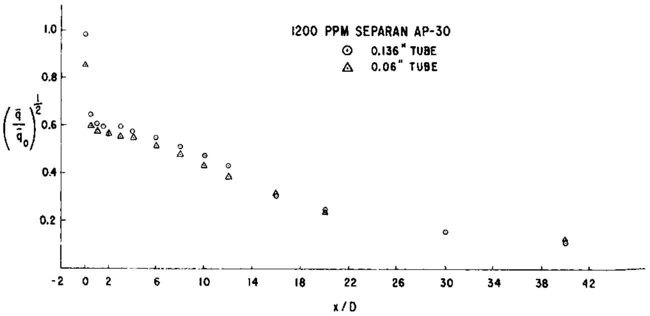

Fig. 4. Pitot tube measurements of axial velocity in a jet of viscoelastic fluid effect of tube size.

TABLE 2. AXIAL MOMENTUM BALANCE FOR 1,200 P.P.M. SEPARAN AP-30 SOLUTION

x/D	Axial momentum (arbitrary units)
0.17	1,126
0.50	1,035
1.0	994
5.0	1,602
10.0	3,245
15.0	4,107
20.0	3,857
30.0	2,438
40.0	1,610

should be independent of position along the jet axis. While it is constant for water, it is not constant for the viscoelastic fluid, as can be seen from the data given in Table 2 for the 1,200 p.p.m. solution. Thus, it is evident that the hot-film anemometer performs differently in viscoelastic fluids than it does in water.

The results obtained with the hot-film anemometer in the 1,200 p.p.m. Carbopol 934 solution are presented in Figure 6. Clearly, the results are not very different from those obtained with water, in spite of the very different shear characteristics of the two fluids. In fact, the shear characteristics of the Carbopol solution are similar to the 600 p.p.m. solution of Separan AP30. Thus, it seems reasonable to attribute the anomalous behavior in the Separan solutions to the elastic properties (which certainly increase with concentration) of the material and not to any effect of a shear dependent viscosity.

DISCUSSION

The cause of the anomalous behavior in the viscoelastic fluids has tentatively been assigned to the effect of turbulence on the rate of heat transfer from the film probe. In this study, the probe was calibrated at the origin of the flow, where there is essentially no turbulence. (A linear relation between voltage and velocity was obtained by means of a linearizing circuit, the velocity being calculated from the pressure measured behind the orifice.) It was then assumed that this calibration remained valid in the turbulent regime. In the case of water, the good agreement between the hot-film and the Pitot tube measurements shows that, within a few percent at least, this assumption was a good one. However, in the case of the viscoelastic fluids, the error in velocity amounts to about (if we assume the correctness of the Pitot tube measurements in the fully developed region of the flow) 20% at 120 p.p.m., 140% at 600 p.p.m., and 250% at 1,200 p.p.m.

The data permits the calculation of the effect of the turbulence on the heat transfer coefficient In Table 3 there is recorded h_t/h_L, the ratio of the heat transfer coefficient in the fully developed mean-flow regime to that in the laminar regime for the different solutions. There is recorded also h_L/h_{Lw}, the heat transfer coefficient for the probe immersed in polymer solution relative to that obtained in water under laminar flow conditions (at the orifice), and h_t/h_{tw}, the transfer coefficient for the polymer solution relative to that for water under conditions of turbulent flow. The approximate range of turbulence intensities over which this effect occurs is indicated.

Kestin, Maeder, and Wang (*13*) and Büyüktür, Kestin, and Maeder (*14*) examined the problem of the effect of free stream turbulence on local heat transfer rates and found that the local coefficient is directly affected only in the case of the laminar boundary layer with a pressure gradient. Under these conditions, an increase is the rate of heat transfer of 5 to 10% occurred at a turbulence intensity of about 5%. In the absence of the pressure gradient, however, it was found that the coefficient is affected only insofar as the increase in intensity hastens the transition from a laminar to a turbulent boundary-layer flow. The existence of disturbances in the laminar boundary layer caused by the free stream turbulence was also demonstrated (*14*). These results, while in agreement with the hot-film measurements in water, do suggest that the anomalous results obtained with the viscoelastic fluids are due to the effect of turbulence. The problem, of course, is that Kestin and his co-workers did not examine turbulence levels anywhere near those found in a free jet flow, and the only fluid studied was air.

FURTHER EXPERIMENTAL EVIDENCE

Other data obtained with the hot-film anemometer also tend to substantiate the hypothesis of the effect of turbulence on the rate of heat transfer from the probe. Figure 7 presents velocity measurements taken along the jet axis in the 600 p.p.m. solution at three different initial velocities: 20, 12, and 4 ft./sec. The turbulence velocities at 20 and 4 ft./sec. are illustrated. It is seen that at 12 and 20 ft./sec., the mean velocity distributions are virtually identical. At 4 ft./sec., however, a laminar jet is formed over a distance of the order of 12 to 14 diam. The absence of any anomalous behavior in this case is evident.

The inverse plot of the mean velocity measurements in water and in the 120, 600, and 1,200 p.p.m. Separan AP30 solutions is presented in Figure 8. It can be seen that the linear relationship (1) is valid for all four fluids in the region of fully developed mean flow, although greater deviations from linearity occur in the 600 and 1,200 p.p.m. solutions. The data do not fall on a single line because the turbulence effect is different for each fluid.

Values for the half-width $r_{1/2}$ were measured as a function of axial position. The linear relationship [Equation 2)] predicted for the fully developed flow regime was obtained (*12*). Although at 1,200 p.p.m. the jet tends to be somewhat narrower then for the case of water (probably due to a delay in the onset of turbulence), once the flow becomes fully developed, the jets all spread at essentially the same rate.

Hot-film measurements of the mean velocity as a function of radial position also showed that the profiles were independent of axial position in the fully developed flow when scaled by an appropriate velocity and length scale. Furthermore, it was found that the data for all four fluids could be fitted by the same normal distribution (*12*).

TABLE 3. EFFECT OF TURBULENCE ON THE HEAT TRANSFER FROM HOT-FILM PROBE

Polymer conc., p.p.m.	% increase in apparent velocity	$\frac{h_t}{h_L}$	$\frac{h_L}{h_{Lw}}$	$\frac{h_t}{h_{tw}}$	Range of turbulence intensity over which effect occurs
0	5	1.0	1.0	1.0	0-2
120	20	1.04	0.94	0.98	5-15
600	140	1.26	0.80	1.01	5-17.5
1,200	250	1.37	0.60	0.82	5-17.5

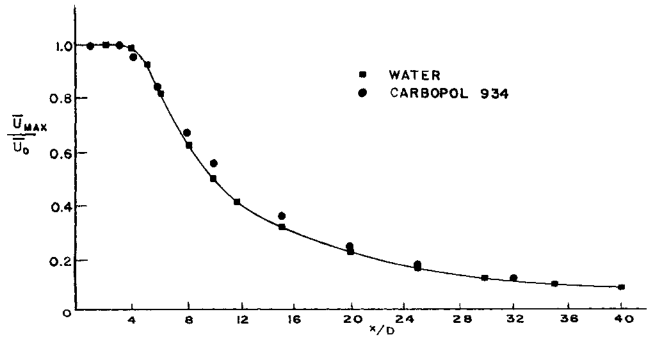

Fig. 6. Axial velocity measured in a water jet and in a 1,200 ppm water solution of Carbopol 934 hot film.

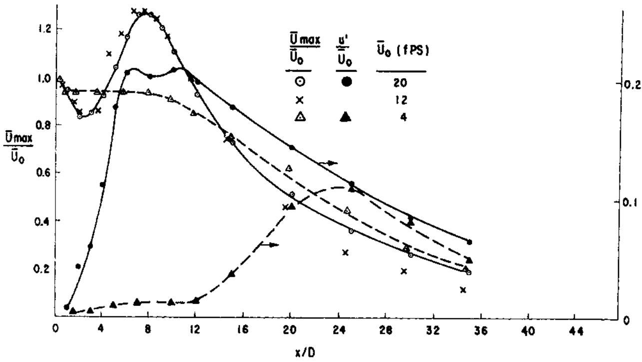

Fig. 7. Hot-film measurements of axial velocity and turbulence intensity in a 600 ppm water solution of Separan AP-30 effect of effux velocity.

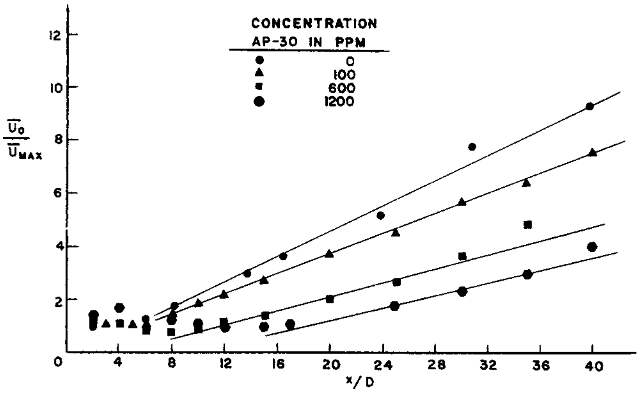

Fig. 8. Axial velocities plotted according to Equation (1).

The above information, together with the Pitot tube data, indicates that the hot-film anemometer is functioning normally in the fully developed region of the jet. Also, it appears that the structure of the mean flow in the viscoelastic fluids is very similar to that in water. [This conclusion is in general agreement with the data of White (*15*), who made Pitot tube measurements in various polymer solutions.] These facts suggest that the effect of the turbulence may be to simply shift the calibration curve for the hot-film probe upwards, and, after a sufficiently high value of the turbulence intensity is reached, it has no further effect on the heat transfer rate. With the assumption that the center-line velocity profiles in the viscoelastic fluid are nearly the same as for water at points sufficiently far downstream, it should then be possible to correct the mean velocity measurements by simply dividing by the appropriate constant factor, that is, $(1 + c)$. The results of this correction are presented in Figure 9. It can be seen that the values are reasonably consistent with the Pitot tube data, particularly those for the 120 p.p.m. solution which are nearly identical with the water curve. At the higher polymer concentrations, the curves fall off somewhat more rapidly than indicated by the Pitot tube measurements, however. A more rapid decrease in these fluids than in water does not seem unreasonable, as some viscoelastic effects are to be expected. Also, the accuracy of the Pitot tube data at these low heads is somewhat poorer in the more concentrated solutions owing to the difficulty of attaining equilibrium during the measurements. On the other hand, it is possible that the effect of the turbulence is also a function of velocity and that the low values are simply the result of an overcorrection.

However, if it is assumed that this correction is valid and that the hot-film measurements in the laminar portion of the jet are correct, then there remains a small region of uncertainty in the curves. In this region, the dashed lines in Figure 9 have been drawn in to provide a smooth transition between the other two regions.

THE EFFECT OF RADIAL TURBULENCE

It has been suggested that the anomalous results obtained with the hot-film anemometer in the non-Newtonian fluids could be due to the presence of large radial and tangential components of turbulence in these fluids. Preliminary measurements of the radial component of turbulence indicate that the values in the viscoelastic jets are not greatly different from those in the water jet. The magnitudes required to produce the observed effects can be estimated. Let Γ be the magnitude of the velocity vector at a given instant of time at a given point in the jet. This is the velocity measured by the anemometer (with the angular dependence of the sensitivity of the probe neglected). Then

$$\overline{\Gamma} = \overline{[(\overline{U} + u)^2 + v^2 + w^2]^{1/2}}$$

where the overbar denotes the time average. Letting

$$v^2 + w^2 = (\alpha - 1)\, u^2$$

one obtains

$$\Gamma = \overline{U}\,\overline{\left[1 + 2\frac{u}{\overline{U}} + \alpha\left(\frac{u}{\overline{U}}\right)^2\right]^{1/2}}$$

Expanding the bracketed quantity, we get

$$\frac{\overline{\Gamma}}{\overline{U}} = 1 + \frac{\alpha}{2}\overline{\left(\frac{u}{\overline{U}}\right)^2} + 0\left[\overline{\left(\frac{u}{\overline{U}}\right)^3}\right]$$

For the 600 p.p.m. solution, $\overline{\Gamma}/\overline{U} = 2.4$, and, using the measured value of $\overline{(u/\overline{U})^2} \approx 0.1$, one obtains $(\alpha - 1) = 27$, if the higher-order terms are neglected. Thus

$$\overline{v^2} + \overline{w^2} = 27\,\overline{u^2}$$

For Newtonian fluids, Corrsin (*16*) found that $\overline{u^2} \approx \overline{v^2}$; similarly, Sami et al. (*17*) found $\overline{u^2} \approx \overline{2v^2} \approx \overline{2w^2}$. Assuming here that $\overline{v^2} \approx \overline{w^2}$, we get

$$\overline{v^2} \approx 14\,\overline{u^2}$$

$$[\overline{v^2}]^{1/2} \approx [14]^{1/2}\,(\overline{u^2})^{1/2} \approx 0.3\,[14]^{1/2}\,\overline{U} \approx \overline{U}$$

Thus, the root-mean-square of the radial turbulence must be of the order of the velocity of the mean flow. Since this value seems unreasonably high, the hypothesis is rejected.

THE EFFECT OF POLYMER ACCUMULATION ON THE PROBE

Lumley has suggested the possibility that the difference in heat transfer between the laminar flow region in which the probe was calibrated and the turbulent region was due to the accumulation of a layer of agglomerated polymer molecules on the surface of the probe in the laminar region, this layer being much thinner or absent in the turbulent region. While this explanation cannot be discounted by the present results, several factors militate against its acceptance in the present work. For example, the curves in Figure 5 were extremely reproducible, and the voltage readings were quite stable. Also, identical results were obtained when traversing from the laminar to the turbulent regime and vice versa. Thus, the processes of accumulation and diminution of the polymer layer

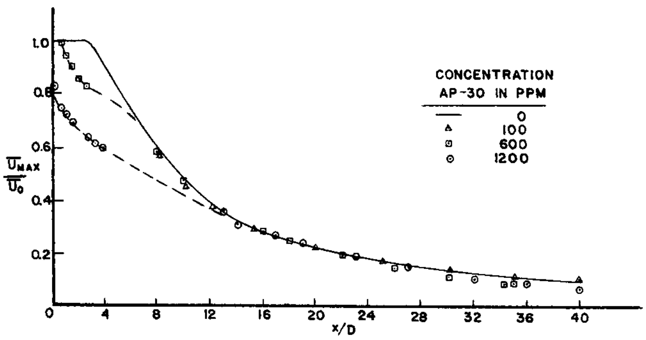

Fig. 9. Corrected hot-film measurements.

would have to be very rapid and entirely reversible. In addition, it is not clear why such a layer would form with the Separan polymer and not with the Carbopol polymer. Finally, spectral measurements in the transition region between laminar and fully developed turbulent flow (*12*) indicate that the frequency response of the probe was not significantly affected by any of the additives.

Thus, a more reasonable explanation of the effect seems to be the turbulence acting across a laminar boundary layer à la Kestin et al. (*13, 14*), together with an alteration in the structure of the boundary layer, perhaps of the form predicted by Metzner and Astarita (*1*), in the viscoelastic fluids. It is clear, however, that further work is needed to resolve this point.

SUMMARY

In summary, the following points may be made. Care must be taken in interpreting results obtained with hot-film wedges (and probably cylinders and cones as well) in flows of viscoelastic fluids when both laminar and turbulent regimes are present. The probes are sensitive not only to the velocity of the fluid but also to the intensity (and probably the scales) of the turbulence. Differences in measured mean velocity as high as 250% above that with water have been observed. Since the results of Kestin et al. (*13, 14*) suggest that this effect may be dependent upon the pressure gradient along the probe (that is, on the shape of the probe), probe design for use in viscoelastic fluids should be considered carefully.

As a final note, it should be recognized that the nature of the turbulence fluctuations producing the wide variations in the heat transfer characteristics of the probe have not been commented upon. From measurements of the turbulence energy spectrum in these flows, the authors have concluded that the fluctuations are not artifacts due to the presence of the probe in the fluid but are, in fact, generated by the flow (*12*).

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NOTATION

a = constant in Equation (1)

c = percent difference between hot-film measurements in the polymer solution and in water

D = orifice diameter

h_L = heat transfer coefficient for probe in laminar regime

h_t = heat transfer coefficient for probe in turbulent regime

h_{Lw} = heat transfer coefficient for probe in water, laminar regime

h_{tw} = heat transfer coefficient for probe in water, turbulent regime

K = parameter in power law fluid model

n = parameter in power law fluid model

P_p = pressure measured with Pitot tube

P_s = stagnation pressure

$\overline{q}$ = mean total head

$\overline{q}_0$ = mean total head at origin of the flow

r = radial direction in the jet

$r_{1/2}$ = half-width of jet

U = instantaneous axial component of velocity

$\overline{U}$ = mean axial component of velocity

$\overline{U}_0$ = mean axial component of velocity at origin of the flow

$\overline{U}_{\max}$ = mean axial component of velocity on jet axis

u = axial component of turbulence velocity

$\overline{u^2}$ = time-averaged axial component of the square of the turbulence velocity

u' = $\sqrt{\overline{u^2}}$

v = radial component of turbulence velocity

w = tangential component of turbulence velocity

x = axial distance measured from the orifice

Greek Letters

α = $1 + (v^2 + w^2)/u^2$

Γ = magnitude of the instantaneous velocity vector in the jet

ρ = fluid density

$\overline{\sigma_x^*}$ = mean axial component of the deviatoric normal stress

τ_{xy} = shear stress in viscometric flow

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